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Understanding Curriculum Alignment: Assessing the Alignment Between State and National Standards

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Executive Summary

Curriculum standards reflect skills that students are expected to master in a particular content area and at a given grade level. Clear articulation of curriculum standards coupled with rigorous assessment lie at the heart of the systemic school initiatives of No Child Left Behind. In 2003 the West Virginia Department of Education (WVDE) created new curriculum standards that were similar to those developed by the Mid-Continental Regional Educational Laboratory (McREL). In 2006, as part of the re-focusing its education system on 21st Century Skills, WVDE revised its curriculum standards to align with those created for the National Assessment for Educational Progress (NAEP). This study analyzes the effect of the WVDE decision to change its curriculum standards. Conducted at the behest of Dr. Steven Paine, WV State Commissioner of Education, we evaluated whether the state accomplished its realignment goal. We limited our analysis to mathematics curricula in the 3rd to 5th grades.

Significance

Curriculum standards are an important topic because they are at the heart of the accountability system developed for No Child Left Behind (NCLB). NCLB invests states with the responsibility to create the standards for proficiency and then to assess students against those standards in the core subjects of mathematics and language arts starting in 3rd grade. If standards are set too low then achieving proficiency may not be an indicator that the student has mastered the skills necessary to take on more advanced topics within the content area or eventually to compete successfully in the labor market nationally or internationally.

National standards such as those articulated by McREL and NAEP are considered high quality and rigorous. McREL created its standards as part of the education community's response to the 1983 Nation at Risk report and follow-up national efforts such as the 1989 education summit. NAEP, which was created by Congress in 1969 and is now often called the Nation's Report Card, assesses students nationally in the 4th, 8th and 12th grade. There is concern both in the States and nationally that the accountability standards from NCLB may be driving down standards. Critics, which include former Secretaries of Education Bennett, Riley, and Paige, point to the large gap in proficiency scores on the NAEP and most state assessments. On average, state assessment scores are about 35 percent higher than 4th grade NAEP scores in mathematics, and in the Appalachian region the gap is even higher. These critics argue that the reason for the gap is that NAEP standards are more rigorous than state standards, and as pressures for NCLB accountability grow, states will be tempted to lower standards further. However, there are a variety of alternative explanations for the proficiency gap-- some technical issues (e.g., whether NAEP is representative at the state level; assessment item construction, format and validity) and some pedagogical or policy. This paper will focus on these latter issues.

Methodology

Alignment studies are not new. However, most alignment studies deal with standards-assessment alignment, that is, do the assessments students take to measure proficiency adequately reflect the content areas articulated in the curriculum standards? This alignment study focuses on standards-to-standards alignment, that is, are the WV standards the same as national standards[1]? Standard-to-standard alignment studies are challenging because standards are typically made up of a series of statements, and different authors may have different styles, vocabularies, and philosophies in articulating these statements. These differences have made it difficult to make comparisons between different sets of standards. To address this problem, the National Science Foundation created the Achievement Standards Network (ASN). The ASN is a large collection of standards, which have been entered into the system following a uniform set of guidelines. The ASN is designed to facilitate comparisons of standards by using algorithms to investigate similarities and differences in the language contained in different standards. To ensure the accuracy of the ASN's mapping of standards, we conducted some validation analyses. We had two independent raters compare statements

[1] In this paper we use the term "national standards" as a general descriptor reflecting a body of diverse standards from a variety of professional and content area organizations. The reasons we focused on McREL and NAEP are that (a) they are widely regarded in the education community as the most rigorous sets of standards; and (b) they are both explicitly used to generate assessments, as are the WV standards.

from a random sample of two different sets of standards to determine whether they matched. We then compared their ratings to the results of the ASN algorithm and we found that in nearly all cases our raters agreed with the ASN algorithm.

Results

This study has three major findings:

First, WVDE has done a good job of aligning both their old and new standards to national standards. In comparing both the WV's 2003 standards for 3rd to 5th grade to the McREL Level II (i.e., 3rd to 5th grade) standards and the WV's 2006 standards for 3rd to 5th grade to the NAEP 4th grade standards, the WV standards covered 100 percent of the performance benchmarks. In both cases, WV standards include even more benchmarks than the national standards so the WV standards are more extensive than recent national standards.

Second, we also found a considerable amount of variation between the McREL standards and the NAEP standards. Only 64 percent of the benchmarks in the NAEP standards are also in the McREL standards. One apparent implication of this finding is that the shift in standards will have an impact on the curriculum as the curriculum focuses on a different set of benchmarks. The McREL-NAEP comparison, however, may exaggerate the change in the curriculum because both the old and new WV standards cover more benchmarks than either McREL or NAEP. Nonetheless more than 8 percent of the benchmarks included in the 2006 standards were not in the 2003 standards.

Third, even with the complete alignment of WV and NAEP standards, there are problems in comparing proficiency scores from state assessments and NAEP. A problem arises because the WV standards cover grades three to five, but the NAEP is administered to fourth graders. A more detailed comparison found that approximately 40 percent of the benchmarks in the NAEP standards do not enter the WV standards until 5th grade.

Discussion

Although this study is narrowly focused on one set of mathematics standards in one state, it raises two issues about the interplay between standards and assessments that are worth further discussion. These issues could have more general significance both for West Virginia and nationally. First, changing standards can have a long-term impact on assessment scores. Curriculum standards define what students are taught. As standards change, students may receive instruction in different concepts. If the new assessments were perfectly aligned to the new standards, as the new assessments are implemented, students may not have been exposed to some of the concepts covered by these new assessments. Thus, when WV's 2006 fifth grade curriculum standards and assessments are implemented in 2008, the first cohort of test takers will have spent most of their academic careers under the old standards—that is, students may not get exposed to some topics covered by the new standards. As each successive cohort spends more time with the new curriculum standards, their actual instruction will become better aligned with the assessment. In the high-stakes environment of NCLB accountability, this lag between actual curriculum exposure and the assessment could reduce proficiency rates, affect the state's ability to reach 100 percent proficiency by 2014, and subject more schools to sanctions.

Second, this analysis potentially adds a new element to the debate about the proficiency gap between state assessments and the NAEP. The scope of WV 2006 third to firth grade standards are fully aligned with the NAEP standards, but they are not aligned in sequence. A considerable percentage of the NAEP standards benchmarks do not enter the WV standards until fifth grade. Thus, WV students taking the fourth grade NAEP will still not have received instruction in some of the material on the test. Under these circumstances lower performance on the NAEP than on the state assessment would be expected. Whether the sequencing of the NAEP or the WV standards is better for learning is a pedagogical issue that is beyond the scope of this paper. Nonetheless, the alignment of WV standards to the NAEP will not fully close the NAEP gap

Introduction

Curriculum standards are lists of skills that students should master in a particular content area and at a given grade. These standards are at the heart of the accountability system created under the No Child Left Behind (NCLB) Act because students are assessed against the standards to determine whether or not they have achieve proficiency—that is, whether or not they have been left behind. NCLB invests the States with responsibility for setting their own curriculum standards for its students, as well as assessing their students' level of proficiency in acquiring the skills outlined in their curriculum standards.

The purpose of this fast response project is to compare standards established for students in West Virginia to national standards in elementary school mathematics—specifically in grades 3 to 5. REL Appalachia conducted this research at the behest of Dr. Steven Paine, WV State School Superintendent. The West Virginia Department of Education (WVDE) is in the process of revising all its curriculum standards with the goal of aligning them with national standards—specifically the standards created for the National Assessment for Educational Progress (NAEP). Congress created the NAEP in 1969 to serve as a periodic benchmark of student performance on a national level to allow for direct comparisons both across states and across years. Under the NAEP program, students are tested in the 4th, 8th, and 12th grades in language arts and mathematics. The National Assessment Governing Board, which administers NAEP, has created (NAEP) standards in each of these subject and grade combination as a foundation for the development of the tests. In the years since NAEP's inception, particularly following the passage of NCLB, they have become increasingly important, and are seen as a way to gauge the rigor of state assessments.

NAEP, however, is not the only articulation of national standards available. Widespread interest in creating and improving rigorous curriculum standards can be traced to the 1983 report "A Nation at Risk" (National Commission on Excellence in Education, 1983). The report was seen as a "wake-up" call for American public education.

Concerned about the viability and long-term health of American public education, in September 1989 President Bush convened the first education summit, which agreed to six broad education reform goals (See Marzano & Kendall, 1998). These goals mandated American educators to “identify rigorous standards regarding what students should know and be able to do in all academic areas.” As a consequence, subject matter and professional organizations started to establish content standards in their respective areas. One of those organizations was the Mid-Continent Research for Education and Learning (McREL). McREL developed a national set of standards for a variety of content areas, and these standards have been widely used as a model for curriculum reform, and they are regarded as a comprehensive national set of curriculum standards. The McREL standards are significant for this paper for two reasons: (a) they are similar to the standards previously in use in West Virginia, which were developed in 2003, and (b) they are also specifically linked to assessment items which states can use in designing their assessments.

In this paper we will make a four-way comparison of elementary school mathematics standards, that is, the 2003 and 2006 standards for 3^{rd} to 5^{th} grade in West Virginia, the 2003 McREL Level II (i.e., 3^{rd} to 5^{th} grades) standards, and the 2006 NAEP 4^{th} grade standards. This comparison will allow for a better understanding of the evolution of West Virginia standards over time. It could also provide insights into possible changes in performance of West Virginia students over time on the NAEP as the classroom instruction they receive becomes better aligned with this national test.

Broader Policy Context

Although this paper will focus on the changing standards in West Virginia and their alignment with national standards, its potential importance goes beyond the narrow issue for one state. There is concern both in States and nationally about the quality of the standards particularly in the core subjects of reading and mathematics. If standards are set too low, achieving proficiency may not really indicate that students are equipped with the skills to master more advanced subjects or eventually to compete successfully in the labor market nationally or internationally. A major exhibit raising these concerns about State standards is the comparison between student performance on the NAEP and on state assessments. Perennially, there has been a large gap between the percentage of students receiving a proficient score on nearly all statewide assessments and those receiving a proficient on the NAEP assessment. For example, for the 2004/2005 Academic Year, the average difference between State and NAEP assessment scores nationwide for Grade 4 Reading was 42 percentage points--that is, on average State assessments scores were 42 percentage points higher than NAEP assessment scores. For Grade 4 math, the average State-NAEP assessment score difference nationwide was 35 percentage points. In the Appalachian region the gaps are even larger (e.g., nearly 60 points on each assessment in Tennessee).

There are a variety of explanations proffered for the gap—some policy and some technical. The policy explanations have focused on the possible relative laxity of state standards. Nearly a decade ago and before the passage of NCLB, then Secretary of Education Richard Riley argued that the NAEP is simply "more challenging than States' own tests" (Riley, 1997). Central to this explanation is that NAEP assesses a similar set of skills, that is, are derived from the same standards, as State assessments, but NAEP has greater depth and is more sensitive to performance differences.

Since the passage of NCLB, there is a growing concern that the law may be driving states to "low-ball" their curriculum standards in a "race to the bottom". The attention and consequences of high-stakes testing places pressure on a State to perform well. States have to achieve certain student performance target goals. One way to maintain high student performance levels in this high-pressure environment is to set the proficiency bar low. This argument was most recently articulated by former Secretaries of Education William Bennett and Rodney Paige (See *Washington Post*, September 21, 2006; A25). Citing a recent report titled *Grading the Systems: The Guide to State Standards, Tests, and Accountability Policies* from the Fordham Foundation (The Fordham Foudation, 2005), Bennett and Paige argue that there is increasing evidence that the states have implemented "mediocre standards," and are "playing games with their tests and accountability systems." Of course, West Virginia's move toward aligning their assessments with the NAEP shows that such a practice, if it exists, is not universal.

The concerns raised by the policy community are not the only possible explanation for the gap between state assessments and the NAEP. A recent report cautions that "A state's performance gap on the two exams should not automatically be interpreted as an indicator of the relative rigor of either test" (See Schoolsmatter.org, 2005). One technical explanation is that although NAEP tests a representative sample of students nationally, it does not necessarily draw a representative sample of each state. For example, in order to have a nationally representative sample, some states including Kentucky and West Virginia have oversampled rural populations. This oversampling could have an impact on the comparison between NAEP and state assessment results. If rural populations, on average, have lower test scores than the rest of the population, NAEP scores would be lower than the state assessment scores just because of the oversampling. Although this oversampling might explain the above average gap for the States in the Appalachia region, it cannot explain the gap in nearly all States.

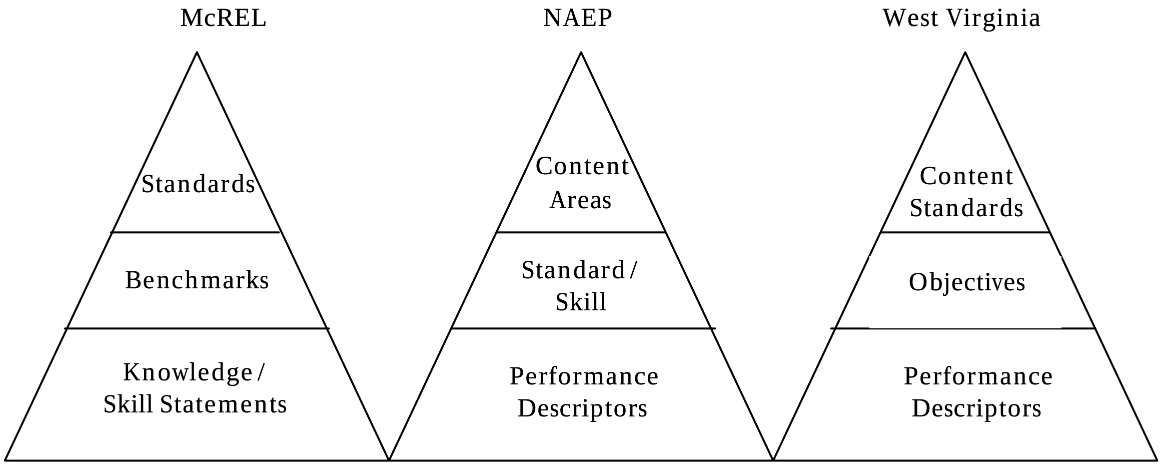
Another technical explanation is that as NAEP adjusts its standards, States will by definition lag behind because it takes time for the States to adjust the standards themselves and then implement the new standards in the classroom. For example, the new WV standards and assessments will not be implemented statewide until 2008. If entering 4th grade students have been receiving instruction under an old set of standards, they may not have the cumulative skills to be able to learn all the skills required under the new standards prior to taking the NAEP. Thus, in WV students may be taking the NAEP without having received instruction in all the material covered by the test. Our strategy of using a four-way comparison among NAEP, McREL, and the two sets of West Virginia standards should provide some insights on this issue. We may find, for example, that West Virginia has taught some concepts in the fifth grade that the NAEP assumes is taught by the fourth grade. The difference in timing may be caused by lower expectations for mastery throughout K-12 education or it may be caused by pedagogical differences about the sequence and timing in the presentation of concepts. The consequence of the difference would likely be lower NAEP scores than state assessments scores, but the differences would not necessarily mean a laxity of standards.

It is the four-way comparison planned in this paper that will provide insights into this latter explanation. If West Virginia's 2003 standards are well aligned with the McREL 2003 but the McREL and NAEP standards are not well aligned, then students in West Virginia may not have been learning some of the material on the NAEP exam, and so it is not surprising that they are performing poorly on the NAEP relative to the state assessment. As West Virginia brings its classroom instruction in line with its new standards, and these standards are well aligned with the NAEP, then a closing of the gap between the NAEP and state assessment scores could be expected in the future.

What Are Curriculum Standards?

Our analysis will compare curriculum standards from three different sources: McREL, NAEP, and the West Virginia Department of Education. Although each set uses slightly different language, all three sets of standards are developed within a hierarchical framework that includes three levels. Because of the differences in language, it is important for this analysis to specify the relationship among each set of terms for each source. Figure 1 depicts the hierarchical classifications for each of the three sources.

Figure 1: Hierarchical Classifications of Under Sources of Standards



At the top of the hierarchy under the McREL classification scheme are Standards, which are common across all ages and grade levels. Each McREL Standard consists of a series of Benchmarks, which vary by grade-level groups. The benchmarks are set for each of five levels corresponding to a set of grades (i.e., pre-K; Level I (K-2), Level II (3-5), Level III (6-8), and Level IV (9-12)). In order to demonstrate proficiency the standard also has a series of performance indicators, called Knowledge or Skill. These performance indicators define what should be taught in the classroom, i.e., curriculum. Furthermore, these indicators are linked to assessment items. Table 1 below shows an excerpt from one of the Level II benchmarks for the McREL Mathematics standards. For example, benchmark 1, "Using a variety of strategies in problem solving", has eight associated Knowledge or Skill statements. These latter statements add detail and context to the benchmarks, and they can be used to develop assessment questions and curricula materials.

Both NAEP and the West Virginia Department of Education use different language in their hierarchies, but these hierarchies are similar nonetheless. Because of the potential complexity of the language in moving among these hierarchies, for the remainder of this paper we will use the McREL terminology in describing the three parts of the hierarchical structure. Thus, even though NAEP uses the language of Content Area, Standard/Skill and Performance Descriptors and West Virginia uses Content Standards, Objectives, and Performance Descriptors, we will use the terms standards, benchmarks and Knowledge/Skill Statements throughout the remainder of the paper.

Several authors have noted the importance of curriculum alignment within the broader context of school reform (La Marca, 2001; Necochea & Cline, 2000; Leitzel & Vogler, 1994). Furthermore, alignment has been highlighted as a mechanism through which one can improve both the efficiency and effectiveness of the educational system (Webb, 1997). However, the overall majority of research on curriculum alignment has focused on either the policy implications of improving alignment (Aviles, 2001; Anderson, 2002); or practical applications and methods to improve curriculum alignment (La Marca, Redfield, & Winter, 2000; Liebling, 1997) between a given State curriculum standard and the assessment derived from said standard.

Table 1. Level II (Grades 3-5) McREL Curriculum Standards Benchmarks and Knowledge/Skill Statements

Benchmark 1:

Uses a variety of strategies in the problem-solving process

Knowledge/Skill Statement 1:

1. Uses a variety of strategies to understand problem situations (e.g., discussing with peers, stating problems in own words, modeling problem with diagrams or physical objects, identifying a pattern)
2. Represents problems situations in a variety of forms (e.g., translates from a diagram to a number or symbolic expression)
3. Understands that some ways of representing a problem are more helpful than others
4. Uses trial and error and the process of elimination to solve problems
5. Knows the difference between pertinent and irrelevant information when solving problems
6. Understands the basic language of logic in mathematical situations (e.g., "and," "or," "not")
7. Uses explanations of the methods and reasoning behind the problem solution to determine reasonableness of and to verify results with respect to the original problem
8. Understands basic valid and invalid arguments (e.g., counter examples, irrelevant approaches)

Benchmark 2:

Understands and applies basic and advanced properties of the concepts of numbers

Knowledge/Skill Statement 2:

1. Understands that numerals are symbols used to represent quantities or attributes of real-world objects
2. Counts whole numbers (i.e., both cardinal and ordinal numbers)
3. Understands symbolic, concrete, and pictorial representations of numbers (e.g., written numerals, objects in sets, number lines)
4. Understands basic whole number relationships (e.g., 4 is less than 10, 30 is 3 tens)
5. Understands the concept of a unit and its subdivision into equal parts (e.g., one object, such as a candy bar, and its division into equal parts to be shared among four people)

Benchmark 3:

Uses basic and advanced procedures while performing the processes of computation

Knowledge/Skill Statement 3:

1. Multiplies and divides whole numbers
2. Adds, subtracts, multiplies, and divides decimals
3. Adds and subtracts simple fractions
4. Uses specific strategies (e.g., front-end estimation, rounding) to estimate computations and to check the reasonableness of computational results
5. Performs basic mental computations (e.g., addition and subtraction of whole numbers)
6. Determines the effects of addition, subtraction, multiplication, and division on size & order of numbers
7. Solves real-world problems involving number operations (e.g., computations with dollars and cents)
8. Knows the language of basic operations (e.g., "products," "multiplication")

Benchmark 4:

Understands and applies basic and advanced properties of the concepts of measurement

Knowledge/Skill Statement 4:

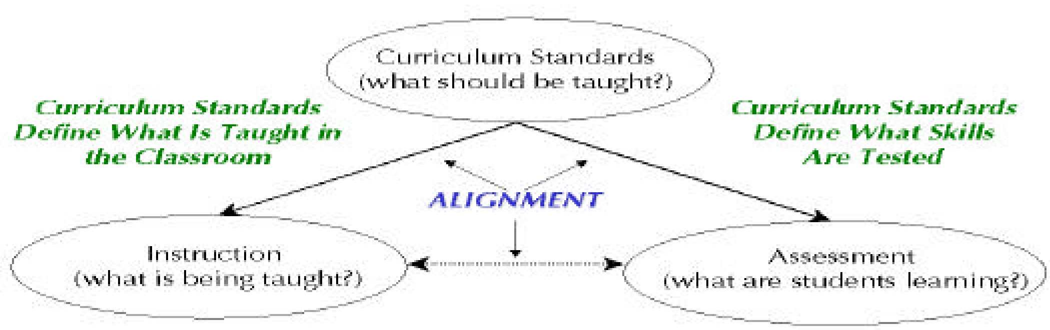
1. Understands the basic measures perimeter, area, volume, capacity, mass, angle, and circumference
2. Selects and uses appropriate tools for given measurement situations (e.g., rulers for length, measuring cups for capacity, protractors for angle)
3. Knows approximate size of basic standard units (e.g., centimeters, feet, grams) and relationships between them (e.g., between inches and feet)
4. Understands relationships between measures (e.g., between length, perimeter, and area) 5. Understands that measurement is not exact (i.e., measurements may give slightly different numbers when measured multiple times)
5. Uses specific strategies to estimate quantities and measurements (e.g., estimating the whole by estimating the parts)
6. Selects and uses appropriate units of measurement, according to type and size of unit

There has been relatively little empirical attention paid to quantifying either (a) the rigor of a given State's curriculum standards; or, (b) the impact of improving curriculum alignment on a school or school systems' efficiency or effectiveness. Furthermore, there is little research on the impact of improving curriculum alignment on student performance. Future research should prospectively assess the impact of improving curriculum alignment on school efficiency, effectiveness, and student achievement.

Why are standards important?

This paper quantifies the extent of alignment between different sets of curriculum standards. Before proceeding with the analysis, it will be useful to put this analysis in perspective. Figure 2 below shows the alignment triangle, that is, the three critical components of the educational system: standards, assessment, and instruction. Standards are the foundational component because they set the expectation for what subjects should be taught and how their learning should be assessed--that is, instruction and assessment are both derived from curriculum standards. If standards are not aligned with the assessment then the assessment does not truly test a students' mastery of the standards. If standards are not aligned with instruction, then students are not taught the material they are expected to master. Misalignment in any of these cases produces poor information about the actual proficiency level of the students.

Figure 2: Overview of the Components of Alignment

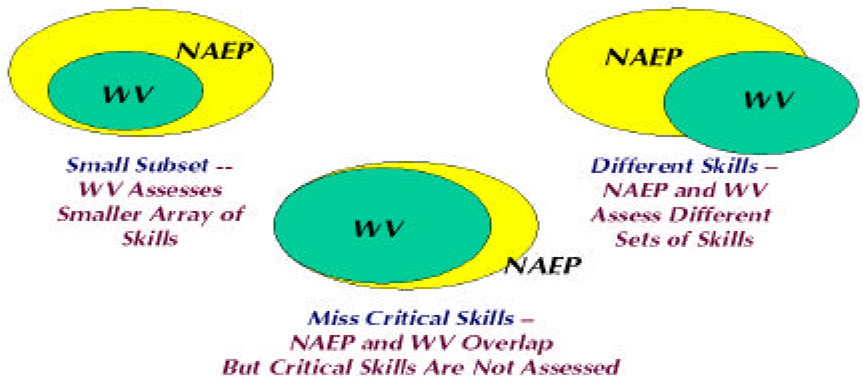


Traditionally, alignment studies (e.g., Webb) have focused on the various connections among the points on this alignment triangle. This study addresses a more fundamental question, that is, are the standards rigorous enough to ensure that proficiency is meaningful. If standards are not high quality, then alignment with assessments or instruction with them is in a sense meaningless because the system is still not accomplishing its goal.

How are State Standards Related to National Standards?

NAEP is widely acknowledged as a rigorous assessment derived from an equally rigorous set of curriculum standards. Having the new WV standards aligned with NAEP would provide presumptive evidence that the new WV standards are indeed rigorous. The figure below shows several possible outcomes for this alignment analysis. Each oval depicts a set of standards. The ovals in the upper left hand portion of the figure show a situation where the NAEP standards are more extensive than the WV standards. This situation is consistent with the criticisms of former Secretaries of Education Bennett, Riley, and Paige that state standards are not as rigorous or extensive as national standards. The ovals in the lower center show a much larger degree of overlap. In this case the state standards are roughly comparable to the NAEP but may be missing a few critical skills. The third set of ovals suggests quite a different situation. In this case there is a partial disagreement about what is important because although there is some overlap of material, the state standard includes some concepts or topics not included in NAEP. In this third situation, it is at least theoretically possible that the state standards could include some rigorous topics not included in the NAEP.

Figure 3: Possible Overlap Between State and National Standards



A Method for Assessing Curriculum Alignment

In order to conduct our curriculum alignment analysis we used the Achievement Standards Network (ASN). As a part of the growing recognition of the importance of the standards to improve the quality and rigor of education, the National Science Foundation created the ASN. The purpose of the ASN is to collect and catalogue standards documents. The ASN currently includes about 1500 documents.

Documents are entered into the ASN system through the collaboration between the ASN staff[2] and the originating organization. They break down each document into a series of individual statements. Each statement is then assigned an ID number similar to an ISDN code. The same statement in different documents will get assigned the same code. Because not all statements use exactly the same language, there is some judgment as to whether two statements are the same or different. These codes allow for the mapping of each document into a series of ID numbers. These numbers then serve as the basis for comparison between different documents or different sets of standards.

In our analysis we calculate the percentage of matching ID numbers as a measure of alignment. For example, suppose document A includes 80 ID numbers and document B includes 60. If 50 of the ID numbers across the two documents are the same, then A is 63 percent aligned with B, and B is 83 percent aligned with A.

One key assumption in this analysis is that the assignment of ID numbers across documents is the same. In order to validate the ASN results we had two raters review half of the West Virginia 2006 standards against the NAEP 2005 standards. Our two raters agreed 95 percent of the time on whether statements in the NAEP document were also in the West Virginia document, and with the ASN matching 96 percent of the

[2] The Achievement Standards Network (ASN) is a joint initiative of the University of Washington, iSchool, JES & Company, and Align-to-Achieve.

time. We took this level of agreement as validating the results of the ASN analysis.

The Four Way Comparison

The analysis in this paper investigates the alignment among the four sets of curriculum standards described above: the West Virginia standards for 2003 and 2006, the McREL standards and the NAEP. The basis for this assessment is the benchmark-like statements included in each of the four sets of standards. The number of benchmark statements varies across the four sets of standards. The West Virginia standards specifically articulate benchmarks for each grade level. Therefore, they tend to be more verbose than the national standards. The 2003 standards have a total of 124 benchmarks. During the revision process for the 2006 some of the standards were changed, but even more were consolidated. The 2006 standards have 93 benchmarks. In contrast, the McREL standards articulate one set of standards encompassing grades 3 through 5, totaling 57 benchmarks, and NAEP only articulates one set of standards for grade 4, totaling 65 benchmarks.

The results of the analysis are summarized in Table 2. The numbers in the table represent the percentage of the statements in the row headings that overlap with the column heading standards divided by the total number of statements in the column heading standards. Thus, the McREL standards included 57 benchmark statements and the West Virginia 2003 standards include 124 benchmark statements. All 57 of the McREL benchmarks map to a benchmark in the West Virginia standards so the percentage shown in the first entry of the second column is 100. Because the West Virginia 2003 standards include so many more statements than the McREL standards, the 57 McREL benchmarks represent only 46 percent of the WV 2003 benchmarks. The second entry in the first column shows this number.

Table 2. Alignment of Four Sets of Curriculum Standards

Standards	Percent Included in Standards			
	WV 2003	McREL	WV 2006	NAEP
WV 2003	--	100	86	57
McREL	46	--	28	64
WV 2006	92	46	--	100
NAEP	30	74	70	--

Our analysis shows that the West Virginia standards for 2003 include all of benchmarks in the McREL standards, and that the revised 2006 standards include all the benchmarks in the current NAEP standards. Thus at this level both sets of standards are aligned with a set of national standards.

This table provides us with the information necessary to answer the question posed by Commissioner Paine of whether the new West Virginia standards are aligned with NAEP. The third entry in the fourth column shows that all of the NAEP benchmarks are included in the West Virginia standards The WV 2006 standards actually include more benchmarks than the NAEP. The fourth entry in the third column shows that the NAEP covers only 70 percent of the new West Virginia benchmarks. Although this information would seem to satisfy the question, as we will discuss below, the analysis is complicated by the fact that the NAEP standards are for fourth grade and the West Virginia standards cover third through fifth grade.

This basic alignment work, however, does not tell the entire story of alignment. Several other interesting questions arise. First, it appears that NAEP and McREL, both national standards, are not that well aligned. This suggests that the shift from a McREL-like set of standards to a NAEP set of standards could have a larger impact on assessments and classroom instruction. As reported in the table, the degree of overlap between the two sets of national standards is less than 75 percent--that is, NAEP includes 74 percent of the McREL benchmarks, and McREL includes only 64 percent of the NAEP benchmarks. Although these sets of standards are considered national models, our alignment analysis suggests that there is not full agreement even among national experts on what constitutes the appropriate set of benchmarks. This result is somewhat surprising because the McREL standards list as one of their core foundational curriculum documents the NAEP Frameworks.

Fourth Grade NAEP and West Virginia's New Standards

The revision of the West Virginia standards was intended to better align those standards with the NAEP because NAEP standards are considered high quality and rigorous. As discussed above this realignment effort has largely succeeded in that the new WV standards now include all the performance descriptors (i.e., benchmarks) of NAEP. One possible consequence of this complete alignment is an improvement among WV students on the fourth grade NAEP resulting in a closing of the gap between NAEP proficiency rates and WV assessment proficiency rates. However, there is a complication that may prevent such an improvement. Because the WV 2006 covers grades three to five but the NAEP is given in 4^{th} grade, some portion of the benchmarks in the West Virginia standards may not be covered by the time of the NAEP.

We have further refined our analysis to separate out the WV benchmarks by grade. We found that only 57 percent of the NAEP performance descriptors (PD) are included or started in either the 3^{rd} or 4^{th} grade standards for West Virginia. Table 3 summarizes the results of this analysis. The table shows the number of performance descriptors in each of NAEP's five content areas (i.e., standards), and how those performance descriptors are distributed among the 3^{rd}, 4^{th}, and 5^{th} grades in the WV 2006 standards. Of the 65 NAEP performance descriptors, parts of 28 are distributed among the three grades and all of 28 fall entirely in the 5^{th} grade.

Table 3. Distribution of NAEP Performance Descriptors (PD) in WV 2006 Standards by Grade Taught

		PD Taught in Grade(s)				
Content Area	PD	3rd Only	4th Only	5th Only	3rd/4th	3rd-5th
Numbers and Properties	20	2	1	3	2	12
Measurement	10	0	0	1	3	6
Geometry	15	0	0	7	0	8
Data Analysis and Probability	9	0	0	9	0	0
Algebraic Representation	11	0	1	8	0	2
Total	65	2	2	28	5	28

The table shows a clear pattern of how the West Virginia 2006 standards intersect with the NAEP standards. West Virginia's new standards spend more time on the foundational topics—that is, numbers and properties, and measurement than seem to be implied by the NAEP standards. About half of the geometry performance descriptors start in 3rd grade and continue until 5th while the rest are not introduced until 5th grade. In the areas of data analysis and probability, and algebraic representation NAEP expects this material to be covered in the 4th grade or before, but West Virginia does not plan to include this material until the 5th grade.

One conclusion of this analysis is that even though West Virginia has adopted the overall goal of following NAEP content standards, it has a different strategy from NAEP for achieving that goal. In particular, West Virginia intends to take an extra year of schooling to present the NAEP content to its students. It is beyond the scope of this paper to evaluate the pedagogical appropriateness of these two approaches, but West Virginia's adoption of a different strategy has both pedagogical and policy implications.

Shifting West Virginia Standards

Although the focus of this paper is on comparisons between the West Virginia and national standards, it is worthwhile considering how West Virginia standards have actually changed between the 2003 version and the 2006 revisions. As mentioned previously, the new standards include about 25 percent fewer benchmarks than the previous standards. Some of these reductions were achieved through consolidation of several 2003 benchmarks into a smaller number of 2006 benchmarks. Others were achieved by deleting some of the 2003 benchmarks altogether. In addition, the 2006 benchmarks include some new statements that were not a part of the 2003 standards at all. Table 4 shows the results of a detailed analysis.

Table 4. Comparison of WV 2003 to WV 2006 Benchmarks by Standards

	Same/ Modified	Consolidated To 2006	Consolidated From 2003	Unique 2003	Unique 2006
Standard 1: Numbers & Operations					
Grade 3	12	4	2	0	0
Grade 4	7	9	3	2	0
Grade 5	9	2	1	1	1
Standard 2: Algebra					
Grade 3	5	0	0	1	0
Grade 4	3	0	0	2	1
Grade 5	3	0	0	1	1
Standard 3: Geometry					
Grade 3	6	2	1	0	0
Grade 4	4	4	2	0	1
Grade 5	3	2	1	1	1
Standard 4: Measurement					
Grade 3	1	7	3	2	1
Grade 4	3	4	1	1	0
Grade 5	8	0	0	1	0
Standard 5: Data Analysis & Probability					
Grade 3	2	0	0	2	1
Grade 4	3	0	0	2	0
Grade 5	2	2	1	1	0
Total	71	36	15	17	7
Percentage of 2003 Benchmarks*	57%	29%		14%	
Percentage of 2006 Benchmarks*	76%		16%		8%

* WV 2003 includes 124 total benchmarks and WV 2006 includes 93 total benchmarks.

Of the total benchmarks, 71 are exactly or roughly the same between the 2003 and 2006 standards. Because the standards included different numbers of benchmarks in different years, this unchanged group represents 57 percent of the 2003 benchmarks and 76 percent of the 2006 benchmarks. Across all the standards, 36 of the benchmarks in 2003 were consolidated into 15 benchmarks in 2006. For example, consider the specific changes to the WV grade 3 performance benchmarks for Curriculum Standard 4: Measurement. The ten benchmarks in the 2003 standards were consolidated into six performance benchmarks in the 2006 standards. Table 5 shows one of the consolidations where three separate 2003 performance benchmarks are focused into a single performance benchmark in the 2006 standards. Additional changes included the deletion of two 2003 performance benchmarks (3.4.2 estimate and count the number of cubes in a rectangular solid to determine volume and MA.3.4.4 understand appropriate grade level conversions within a system of measure), and the inclusion of one new performance benchmark (3.4.2 estimate and find the perimeter and area of familiar geometric shapes, using manipulatives, grids, or appropriate measuring tools).

Table 5. An Example of the Consolidation of Performance Benchmarks from 2003 to 2006

	2003 Curriculum Standards	2006 Curriculum Standards
Standard 4: Measurement	MA.3.4.1 estimate, measure, compare, order and draw lengths using inches (to the nearest ½ inch), feet, yards, centimeters and meters.	M.O.3.4.1 estimate, measure, compare, and order common measurements of objects: length using customary and metric (to the nearest 1/2 inch) temperature in Celsius and Fahrenheit mass/weight
	MA.3.4.5 estimate and measure results of mass/weight in ounces, pounds, grams, and kilograms.	
	MA.3.4.10 estimate, read, and recognize common temperatures of Celsius and Fahrenheit.	

Comparing the McREL Standards to NAEP

Our analyses showed a 63% alignment between the McREL and NAEP standards. Given that both sets of standards are widely seen as reflecting on a general consensus about what skills children should learn and when, this result is somewhat surprising. Furthermore the McREL standards document, as one of their core foundational curriculum documents, specifically lists the 2005 NAEP Mathematics Framework. Table 6 shows the percentage of each of the McREL standards that are derived from the NAEP. Overall only 50% of the McREL performance indicators come from NAEP. Also, the crosswalk between the two ranges widely across the nine McREL standards. The first (problem-solving) and last (the nature of mathematics) standards are totally different from the NAEP, but the eight McREL standard largely comes from the NAEP framework.

Table 6. Percentage of McREL Performance Benchmarks derived from NAEP Frameworks

McREL Standards and Benchmarks (Level II -- Grades 3-5)	Indicators Per Bench-mark	Derived from NAEP Frameworks
Standard 1. Uses a variety of strategies in the problem-solving process	20	0%
Standard 2. Understands and applies basic and advanced properties of the concepts of numbers	28	46%
Standard 3. Uses basic and advanced procedures while performing the processes of computation	50	52%
Standard 4. Understands and applies basic and advanced properties of the concepts of measurement	46	63%
Standard 5. Understands and applies basic and advanced properties of the concepts of geometry	29	66%
Standard 6. Understands and applies basic and advanced concepts of statistics and data analysis	26	42%
Standard 7. Understands and applies basic and advanced concepts of probability	15	47%
Standard 8. Understands and applies basic and advanced properties of functions and algebra	15	88%
Standard 9. Understands the general nature and uses of mathematics	7	0%
Total Number of Performance Indicators for All Standards/Benchmarks	238	

McREL Standards and Benchmarks (Level II – Grades 3-5)	Indicators Per Bench-mark	Derived from NAEP Frameworks
Total Number of Performance Indicators for All Standards/Benchmarks Derived From the NAEP Frameworks	120	
Percent of McREL Performance Indicators Derived From the NAEP Frameworks	50%	

Results

This study has three major findings:

First, WVDE has done a good job of aligning both their old and new standards to national standards. In comparing both the WV's 2003 standards for 3rd to 5th grade to the McREL Level II (i.e., 3rd to 5th grade) standards and the WV's 2006 standards for 3rd to 5th grade to the NAEP 4th grade standards, the WV standards covered 100 percent of the performance benchmarks. In both cases, WV standards covered even more benchmarks than the national standards.

Second, we also found a considerable amount of variation between the McREL standards and the NAEP standards. Only 64 percent of the benchmarks in the NAEP standards are also in the McREL standards. One apparent implication of this finding is that the shift in standards will have a large impact on the curriculum as the curriculum focuses on a different set of benchmarks. The McREL-NAEP comparison, however, may exaggerate the change in the curriculum because both the old and new WV standards cover more benchmarks than either McREL or NAEP. Nonetheless more than 7 percent of the benchmarks included in the 2006 standards were not in the 2003 standards.

Third, even with the complete alignment of WV and NAEP standards, there are problems in comparing proficiency scores from state assessments and NAEP. A problem arises because the WV standards cover grade three to five, but the NAEP is administered to fourth graders. A more detailed comparison found that nearly 40 percent of the benchmarks in the NAEP standards are part of the fifth standards in WV.

Discussion

Although this study is narrowly focused on one set of mathematics standards, it raises two issues about the interplay between standards and assessments that are worth further discussion. These issues could have more general significance both for West Virginia and nationally. First, changing standards can have a long-term impact on assessment scores. Curriculum standards define what students are taught. As standards change, students may receive instruction in different topics. If the new assessments were perfectly aligned to the new standards, as the new assessments are implemented, students may not have been exposed to some of the concepts covered by these new assessments. Thus, when WV's 2006 fifth grade curriculum standards and assessments are implemented in 2008, the first cohort of test takers will have spent most of their academic careers under the old standards. Thus, they may not get exposed to some topics covered by the new standards. As each successive cohort spends more time with the new curriculum standards, their actual education becomes better aligned with the assessment. In the high-stakes environment of NCLB accountability, this lag between actual curriculum exposure and the assessment could reduce proficiency rates and subject more schools to sanctions.

Second, this analysis potentially adds a new element to the debate about the proficiency gap between state assessments and the NAEP. The scope of WV 2006 third to firth grade standards are fully aligned with the NAEP standards, but they are not aligned in sequence. A considerable percentage of the NAEP standards benchmarks do not enter the WV standards until fifth grade. Thus, WV students taking the fourth grade NAEP will still not have been exposed to some of the material on the test. Under these circumstances lower performance on the NAEP than on the state assessment would be expected. Whether the sequencing of the NAEP or the WV standards is better for learning is a pedagogical issue that is beyond the scope of this paper. Nonetheless, the alignment of WV standards to the NAEP will not fully close the NAEP gap.

Whether the sequencing of the NAEP or the WV standards is better for learning is a pedagogical issue that is beyond the scope of this paper, but this paper has helped to frame the central pedagogical issue. The creation of curriculum standards is complicated by two competing pedagogical concerns. First, what skills does one introduce (and when)? For example, does one introduce *probability* in the 3rd, 4th, or 5th grade? Second, there is an inherent tension between building upon the foundational skills laid in the early grades, and the introduction of new and more complex material/skills. That is, to what extent are curriculum standards from grade-to-grade redundant? As noted previously, one difference between national and state curriculum standards is the number of performance benchmarks. In this specific case, the number of performance benchmarks articulated for WV students is considerably more than those in both comparable sets of national standards. One reason for this is that national standards tend to focus on a smaller number of key content areas, but assess those content areas in greater depth. The obvious question that arises is: does having more performance benchmarks strengthen or weaken a set of curriculum standards? While the answer to this question lies beyond the scope of this paper, the answer would appear to lie not in the relative number of performance benchmarks for a given curriculum standard, but in the clarity and specificity of the standard and associated benchmarks themselves. A recent report of the National Council of Teachers of Mathematics (NCTM: 2006) noted that in general mathematics standards are “a mile wide and an inch deep” in that they cover too many topics in superficial detail. Furthermore, Reys et. al. have noted that from state to state there is little consensus on the placement or emphasis of topics within specific grade levels within mathematics curriculum standards (Reys et al. 2005; Reys et al. 2006). They recommended covering a smaller number of “focal” concepts in more depth and rigor. The Fordham Foundation report (2005) described the 2003 West Virginia standards as "lengthy, repetitive, and unwieldy" and argued that regardless of the number of standards and benchmarks, standards should be clear and specific academic standards, actionable by teachers, and understandable by parents.

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Appendix A: McREL 4th Edition Level II (Grades 3-5) Curriculum Standards

Benchmarks	Level II (Grades 3-5) – Knowledge/Skill Statements
Standard 1. Uses a variety of strategies in the problem-solving process	1. Uses a variety of strategies to understand problem situations (e.g., discussing with peers, stating problems in own words, modeling problem with diagrams or physical objects, identifying a pattern)
	2. Represents problems situations in a variety of forms (e.g., translates from a diagram to a number or symbolic expression)
	3. Understands that some ways of representing a problem are more helpful than others
	4. Uses trial and error and the process of elimination to solve problems
	5. Knows the difference between pertinent and irrelevant information when solving problems
	6. Understands the basic language of logic in mathematical situations (e.g., "and," "or," "not")
	7. Uses explanations of the methods and reasoning behind the problem solution to determine reasonableness of and to verify results with respect to the original problem
	8. Understands basic valid and invalid arguments (e.g., counter examples, irrelevant approaches)
Standard 2. Understands and applies basic and advanced properties of the concepts of numbers	1. Understands that numerals are symbols used to represent quantities or attributes of real-world objects
	2. Counts whole numbers (i.e., both cardinal and ordinal numbers)
	3. Understands symbolic, concrete, and pictorial representations of numbers (e.g., written numerals, objects in sets, number lines)
	4. Understands basic whole number relationships (e.g., 4 is less than 10, 30 is 3 tens)
	5. Understands the concept of a unit and its subdivision into equal parts (e.g., one object, such as a candy bar, and its division into equal parts to be shared among four people)

Performance Standard	Level II (Grades 3-5) -- Performance Indicators
Standard 3. Uses basic and advanced procedures while performing the processes of computation	1. Multiplies and divides whole numbers
	2. Adds, subtracts, multiplies, and divides decimals
	3. Adds and subtracts simple fractions
	4. Uses specific strategies (e.g., front-end estimation, rounding) to estimate computations and to check the reasonableness of computational results
	5. Performs basic mental computations (e.g., addition and subtraction of whole numbers)
	6. Determines the effects of addition, subtraction, multiplication, and division on size and order of numbers
	7. Understands the properties of and the relationships among addition, subtraction, multiplication, and division (e.g., reversing the order of two addends does not change the sum; division is the inverse of multiplication)
	8. Solves real-world problems involving number operations (e.g., computations with dollars and cents)
	9. Knows the language of basic operations (e.g., "products," "multiplication")
Standard 4. Understands and applies basic and advanced properties of the concepts of measurement	1. Understands the basic measures perimeter, area, volume, capacity, mass, angle, and circumference
	2. Selects and uses appropriate tools for given measurement situations (e.g., rulers for length, measuring cups for capacity, protractors for angle)
	3. Knows approximate size of basic standard units (e.g., centimeters, feet, grams) and relationships between them (e.g., between inches and feet)
	4. Understands relationships between measures (e.g., between length, perimeter, and area)
	5. Understands that measurement is not exact (i.e., measurements may give slightly different numbers when measured multiple times)
	6. Uses specific strategies to estimate quantities and measurements (e.g., estimating the whole by estimating the parts)
	7. Selects and uses appropriate units of measurement, according to type and size of unit

Performance Standard	Level II (Grades 3-5) -- Performance Indicators
Standard 5. Understands and applies basic and advanced properties of the concepts of geometry	1. Knows basic geometric language for describing and naming shapes (e.g., trapezoid, parallelogram, cube, sphere)
	2. Understands basic properties of figures (e.g., two- or three-dimensionality, symmetry, number of faces, type of angle)
	3. Predicts and verifies the effects of combining, subdividing, and changing basic shapes
	4. Understands that shapes can be congruent or similar
	5. Uses motion geometry (e.g., turns, flips, slides) to understand geometric relationships
	6. Understands characteristics of lines (e.g., parallel, perpendicular, intersecting) and angles (e.g., right, acute)
	7. Understands how scale in maps and drawings shows relative size and distance
Standard 6. Understands and applies basic and advanced concepts of statis-tics and data analysis	1. Understands that data represent specific pieces of information about real-world objects or activities
	2. Understands that spreading data out on a number line helps to see what the extremes are, where the data points pile up, and where the gaps are
	3. Understands that a summary of data should include where the middle is and how much spread there is around it
	4. Organizes and displays data in simple bar graphs, pie charts, and line graphs
	5. Reads and interprets simple bar graphs, pie charts, and line graphs
	6. Understands that data come in many different forms and that collecting, organizing, and displaying data can be done in many ways
	7. Understands the basic concept of a sample (e.g., a large sample leads to more reliable information; a small part of something may have unique characteristics but not be an accurate representation of the whole)
Standard 7. Understands and applies basic and advanced concepts of prob-ability	1. Understands that the word "chance" refers to the likelihood of an event
	2. Recognizes events that are sure to happen, events that are sure not to happen, and events that may or may not happen (e.g., in terms of "certain," "uncertain," "likely," "unlikely")
	3. Understands that when predictions are based on what is known about the past, one must assume that condi-tions stay the same from the past event to the predicted future event
	4. Understands that statistical predictions are better for describing what proportion of a group will experience something (e.g., what proportion of automobiles will be involved in accidents) rather than which individuals within the group will experience something, and how often events will occur (e.g., how many sunny days will occur over a year) rather than exactly when they will occur

	5. Uses basic sample spaces (i.e., the set of all possible outcomes) to describe and predict events
Performance Standard	**Level II (Grades 3-5) -- Performance Indicators**
Standard 8. Understands and applies basic and advanced properties of functions and algebra	1. Recognizes a wide variety of patterns (e.g., basic linear patterns such as [2, 4, 6, 8 . . .] ; simple repeating and growing patterns) and the rules that explain them
	2. Understands that the same pattern can be represented in different ways (e.g., geometrically or numerically; the pattern of numbers [7, 14, 21, 28 . . .] is equivalent to the mathematical relationship 7 Ã— n)
	3. Knows that a variable is a letter or symbol that stands for one or more numbers
	4. Understands the basic concept of an equality relationship (i.e., an equation is a number sentence that shows two quantities that are equal)
	5. Solves simple open sentences involving operations on whole numbers (e.g., ? + 17 = 23)
	6. Knows basic characteristics and features of the rectangular coordinate system (e.g., the horizontal axis is the X axis and the vertical axis is the Y axis)
Standard 9. Understands the general nature and uses of mathematics	1. Understands that numbers and the operations performed on them can be used to describe things in the real world and predict what might occur
	2. Understands that mathematical ideas and concepts can be represented concretely, graphically, and symbolically

Appendix B: NAEP 2005 Grade 4 Mathematics Framework

Content Area	Standards/Skills	Performance Descriptor
Numbers and Properties	Number sense	a) Identify the place value and actual value of digits in whole numbers.
		b) Represent numbers using models such as base 10 representations, number lines, and two-dimensional models.
		c) Compose or decompose whole quantities by place value (e.g., write whole numbers in expanded notation using place value: 342 = 300 + 40 + 2).
		d) Write or rename whole numbers (e.g., 10: 5 + 5, 12 – 2, 2 x 5).
		e) Connect model, number word, or number using various models and representations for whole numbers, fractions, and decimals.
		f) Order or compare whole numbers, decimals, or fractions.
	Estimation	a) Use benchmarks (well-known numbers used as meaningful points for comparison) for whole numbers, decimals, or fractions in contexts (e.g., 1/2 and .5 may be used as benchmarks for fractions and decimals between 0 and 1.00).
		b) Make estimates appropriate to a given situation with whole numbers, fractions, or decimals by: knowing when to estimate, selecting the appropriate type of estimate, including over- estimate, underestimate, and range of estimate, or selecting the appropriate method of estimation (e.g., rounding).
		c) Verify solutions or determine the reasonableness of results in meaningful contexts.
	Number operations	a) Add and subtract: whole numbers, or fractions with like denominators, or decimals through hundredths.
		b) Multiply whole numbers: no larger than two-digit by two-digit with paper and pencil computation, or larger numbers with use of calculator.
		c) Divide whole numbers: up to three-digits by one-digit with paper and pencil computation, or up to five-digits by two-digits with use of calculator.
		d) Describe the effect of operations on size (whole numbers).
		e) Interpret whole number operations and the relationships between them.
		f) Solve application problems involving numbers and operations.
	Ratios and proportional reasoning	a) Use simple ratios to describe problem situations.
	Properties of	a) Identify odd and even numbers.

Content Area	Standards/Skills	Performance Descriptor
	number and operations	b) Identify factors of whole numbers.
		c) Apply basic properties of operations.
		d) Explain or justify a mathematical concept or relationship (e.g., explain why 15 is an odd number or why 7–3 is not the same as 3–7).
Measurement	Measuring physical attributes	a) Identify the attribute that is appropriate to measure in a given situation.
		b) Compare objects with respect to a given attribute, such as length, area, volume, time, or temperature.
		c) Estimate the size of an object with respect to a given measurement attribute (e.g., length, perimeter, or area using a grid).
		d) Select or use appropriate measurement instruments such as ruler, meter stick, clock, thermometer, or other scaled instruments.
		e) Solve problems involving perimeter of plane figures.
		f) Solve problems involving area of squares and rectangles.
	Systems of measurement	a) Select or use appropriate type of unit for the attribute being measured such as length, time, or temperature.
		b) Solve problems involving conversions within the same measurement system such as conversions involving inches and feet or hours and minutes.
		c) Determine appropriate size of unit of measurement in problem situation involving such attributes as length, time, capacity, or weight.
		d) Determine situations in which a highly accurate measurement is important.
Geometry	Dimension and shape	a) Explore properties of paths between points.
		b) Identify or describe (informally) real-world objects using simple plane figures (e.g., triangles, rectangles, squares, and circles) and simple solid figures (e.g., cubes, spheres, and cylinders).
		c) Identify or draw angles and other geometric figures in the plane.
		d) Describe attributes of two- and three-dimensional shapes.
	Transformation of shapes and	a) Identify whether a figure is symmetrical, or draw lines of symmetry.
		c) Identify the images resulting from flips (reflections), slides (translations), or turns (rotations).

Content Area	**Standards/Skills**	**Performance Descriptor**
	preservation of properties	d) Recognize which attributes (such as shape and area) change or don't change when plane figures are cut up or rearranged.
		e) Match or draw congruent figures in a given collection.
	Relationships between geometric figures	a) Analyze or describe patterns of geometric figures by increasing number of sides, changing size or orientation (e.g., polygons with more and more sides).
		b) Assemble simple plane shapes to construct a given shape.
		c) Recognize two-dimensional faces of three-dimensional shapes.
		d) Describe and compare properties of simple and compound figures composed of triangles, squares, and rectangles.
	Position and direction	a) Describe relative positions of points and lines using the geometric ideas of parallelism or perpendicularity.
		b) Construct geometric figures with vertices at points on a coordinate grid.
	Mathematical reasoning	a) Distinguish which objects in a collection satisfy a given geometric definition and explain choices.
Data Analysis and Probability	Data representation	a) Read or interpret a single set of data.
		b) For a given set of data, complete a graph (limits of time make it difficult to construct graphs completely).
		c) Solve problems by estimating and computing within a single set of data.
	Characteristics of data sets	a) Given a set of data or a graph, describe the distribution of the data using median, range, or mode.
		b) Compare two sets of related data.
	Experiments and samples	no specific skills
	Probability	a) Use informal probabilistic thinking to describe chance events (i.e., likely and unlikely, certain and impossible)."
		b) Determine a simple probability from a context that includes a picture.
		c) List all possible outcomes of a given situation or event.
		d) Represent the probability of a given outcome using a picture or other graphic.
Algebraic Rep-	Patterns, rela-	a) Recognize, describe, or extend numerical patterns.

Content Area	Standards/Skills	Performance Descriptor
resentations	tions, and functions	b) Given a pattern or sequence, construct or explain a rule that can generate the terms of the pattern or sequence.
		c) Given a description, extend or find a missing term in a pattern or sequence.
		d) Create a different representation of a pattern or sequence given a verbal description.
		e) Recognize or describe a relationship in which quantities change proportionally.
	Algebraic representations	a) Translate between the different forms of representations (symbolic, numerical, verbal, or pictorial) of whole number relationships (such as from a written description to an equation or from a function table to a written description).
		b) Graph or interpret points with whole number or letter coordinates on grids or in the first quadrant of the coordinate plane.
		c) Verify a conclusion using algebraic properties.
	Variables, expressions, and operations	a) Use letters and symbols to represent an unknown quantity in a simple mathematical expression.
		b) Express simple mathematical relationships using number sentences.
	Equations and inequalities	a) Find the value of the unknown in a whole number sentence

Appendix C: West Virginia 2003 Mathematics Content Standards and Objectives for Grades 3-5

Content Standards	Performance Indicators					
	Grade 3		Grade 4		Grade 5	
Standard 1: Number and Operations (MA.S.1)	MA.3.1.1	read, write, order, and compare numbers to 10,000.	MA.4.1.1	read, write, order, and compare numbers to the millions place.	MA.5.1.1	read, write, order and compare all whole numbers.
	MA.3.1.2	read, write, order, and compare decimals to hundredths with models.	MA.4.1.2	read, write, order, and compare decimals to thousandths with and without models and pictorial representations.	MA.5.1.2	read, write, order and compare all decimals.
	MA.3.1.3	identify place value of each digit utilizing standard and expanded form to 10,000.	MA.4.1.3	identify place value of each digit utilizing standard and expanded form through 1,000,000.	MA.5.1.3	identify place value of each digit utilizing standard and expanded form in any whole number.
	MA.3.1.4	estimate to nearer 10,000 and 1,000 using rounding, benchmarks, and compatible numbers to determine reasonableness of an answer.	MA.4.1.4	estimate to nearer 10,000 using rounding, benchmarks, and compatible numbers and identify over and under estimates to determine reasonableness of an answer.	MA.5.1.4	estimate with whole numbers and decimals, including money, to determine reasonableness of an answer.
	MA.3.1.5	identify fractions as part of a whole/one and as part of a group using models and pictorial representations.	MA.4.1.5	compare and order fractions with like and unlike denominators using pictorial representations	MA.5.1.5	identify and use the divisibility rules of 2, 3, 5, 9 and 10.
	MA.3.1.6	compare and order fractions with like and unlike denominators using concrete models.	MA.4.1.6	add and subtract fractions with like and unlike denominators using pictorial representations.	MA.5.1.6	compare and order fractions, improper fractions and mixed numbers with like and unlike denominators (e.g., greatest common factor, lowest common multiple).

Content Standards	Performance Indicators					
	Grade 3		Grade 4		Grade 5	
	MA.3.1.7	add and subtract fractions with like denominators using concrete models and pictorial representations.	MA.4.1.7	recognize and model equivalent fractions using pictorial representations.	MA.5.1.7	model and write equivalencies of fractions, decimals, percents, and ratios.
	MA.3.1.8	recognize and model equivalent fractions using concrete materials.	MA.4.1.8	model addition and subtraction of mixed numbers without and with regrouping.	MA.5.1.8	add and subtract fractions and mixed numbers.
	MA.3.1.9	recognize and model proper and improper fractions and mixed numbers.	MA.4.1.9	understand the relationship of fractions to decimals using concrete objects and pictorial representations.	MA.5.1.9	model multiplication and division of fractions to solve the algorithm.
	MA.3.1.10	add and subtract 2- and 3-digit whole numbers and money without and with regrouping.	MA.4.1.10	round decimals to the nearest whole, 10th, or 100th.	MA.5.1.10	model multiplication of decimals and division of decimals by a whole number divisor.
	MA.3.1.11	understand multiplication as repeated addition and division as repeated subtraction.	MA.4.1.11	add and subtract decimals to the 1000th place.	MA.5.1.11	develop fluency in addition, subtraction, multiplication and division of whole numbers.
	MA.3.1.12	understand meanings of operations and the relationship between multiplication and division (e.g., identity element of multiplication, commutative property, property of zero, fact families, associative property).	MA.4.1.12	apply the distributive property of multiplication over addition.	MA.5.1.12	solve grade level appropriate story problems using multiple strategies.
	MA.3.1.13	memorize basic multiplication facts 0-5 and the corresponding division facts.	MA.4.1.13	memorize basic multiplication facts and corresponding division facts.		

Content Standards	Performance Indicators					
	Grade 3		Grade 4		Grade 5	
	MA.3.1.14	model multiplication of 2- and 3-digit numbers by a 1-digit number.	MA.4.1.14	multiply 2-and 3-digit numbers by 1- and 2-digit numbers.		
	MA.3.1.15	model division of 2- and 3-digit numbers by a 1-digit number.	MA.4.1.15	divide 2-and 3-digit numbers by 1-and 2-digit numbers.		
	MA.3.1.16	solve grade level appropriate story problems using multiple strategies.	MA.4.1.16	apply the order of operations in solving problems.		
			MA.4.1.17	solve grade level appropriate story problems using multiple strategies.		
			MA.4.1.18	develop fluency in addition and subtraction of all whole numbers.		
Standard 2: Algebra (MA.S.2)	MA.3.2.1	analyze and complete a geometric pattern.	MA.4.2.1	solve problems involving patterns.	MA.5.2.1	explore a variety of patterns with missing elements (e.g., square numbers, powers, triangular numbers, arithmetic sequences).
	MA.3.2.2	use input/output model with grade appropriate functions.	MA.4.2.2	use input/output model with grade appropriate functions.	MA.5.2.2	use input/output model with grade appropriate functions.
	MA.3.2.3	identify and write number patterns of 3's and 4's.	MA.4.2.3	understand the relationship between number patterns and multiples.	MA.5.2.3	write an equation using a variable to solve problems.
	MA.3.2.4	identify and write the rule of a given pattern.	MA.4.2.4	use patterns to predict the nth term.	MA.5.2.4	evaluate an expression given a value for the variable.

Content Standards	Performance Indicators					
	Grade 3		Grade 4		Grade 5	
	MA.3.2.5	write equivalent numerical expressions.	MA.4.2.5	represent the idea of a variable as an unknown quantity using a letter.		
	MA.3.2.6	represent the idea of a variable as an unknown quantity using a symbol.				
Standard 3: Geometry (MA.S.3)	MA.3.3.1	identify basic polygons and their components through decagon.	MA.4.3.1	identify plane figures and their components.	MA.5.3.1	classify and compare polygons.
	MA.3.3.2	identify and describe a cube, rectangular solid, cylinder, cone and pyramid according to the number of faces, edges and vertices.	MA.4.3.2	compare and contrast quadrilateral shapes.	MA.5.3.2	construct a 3-dimensional figure from different views (orthogonal drawings).
	MA.3.3.3	from a plane drawing, construct and identify the solid figure.	MA.4.3.3	describe three-dimensional objects from different perspectives.	MA.5.3.3	measure angles using a protractor.
	MA.3.3.4	identify, determine and draw lines of symmetry.	MA.4.3.4	identify and draw intersecting, parallel, and perpendicular lines.	MA.5.3.4	draw a design with more than one line of symmetry.
	MA.3.3.5	model and describe lines and rays.	MA.4.3.5	draw, label, compare, and classify acute, right, and obtuse angles.	MA.5.3.5	recognize the images of figures after reflections, translations and rotations.
	MA.3.3.6	identify and draw right, obtuse and acute angles.	MA.4.3.6	draw a design with one line of symmetry.	MA.5.3.6	draw a similar figure using a scale.
	MA.3.3.7	given a model, draw an example of a flip, slide and turn (reflection, translation, and rotation).	MA.4.3.7	graph/plot ordered pairs on a one-quadrant grid.		

Content Standards	Performance Indicators					
	Grade 3		Grade 4		Grade 5	
	MA.3.3.8	name the location of a point on a one-quadrant grid.	MA.4.3.8	draw and identify parts of a circle: center point, diameter, and radius.		
Standard 4: Measurement (MA.S.4)	MA.3.4.1	estimate, measure, compare, order and draw lengths using inches (to the nearest ½ inch), feet, yards, centimeters and meters.	MA.4.4.1	estimate, measure, compare, order and draw lengths using customary and metric units.	MA.5.4.1	estimate, measure, compare, order and draw lengths of real objects in parts of an inch up to 1/8 of an inch and millimeters.
	MA.3.4.2	estimate and count the number of cubes in a rectangular solid to determine volume.	MA.4.4.2	determine and compare areas of rectangles and squares by multiplying length and width.	MA.5.4.2	determine and compare area of triangles and parallelograms using appropriate formula.
	MA.3.4.3	discover through modeling the formula for determining the area of a rectangle.	MA.4.4.3	discover through modeling the formula for volume of a rectangular prism.	MA.5.4.3	solve problems using the formulas for determining volume of a rectangular prism.
	MA.3.4.4	understand appropriate grade level conversions within a system of measure.	MA.4.4.4	understand appropriate grade level conversions within a system of measure.	MA.5.4.4	understand the relationship between area and perimeter of a plane figure.
	MA.3.4.5	estimate and measure results of mass/weight in ounces, pounds, grams, and kilograms.	MA.4.4.5	read scales of weight, capacity, and temperature and select appropriate unit.	MA.5.4.5	understand appropriate grade level conversions within a system of measure and apply to problem solving situations.
	MA.3.4.6	read time to 5-minute intervals using analog and digital clocks.	MA.4.4.6	read time to the minute.	MA.5.4.6	evaluate and/or measure the weight/mass of real objects in ounces, pounds, tons, grams, and kilograms.

Content Standards	Performance Indicators					
	Grade 3		Grade 4		Grade 5	
	MA.3.4.7	calculate elapsed time to quarter-hour.	MA.4.4.7	determine elapsed time in hours/ minutes within a 24-hour period.	MA.5.4.7	calculate elapsed time.
	MA.3.4.8	read and write amounts of money to $100.00.	MA.4.4.8	count coins and bills and determine correct change.	MA.5.4.8	select appropriate customary and metric units and the tools for measuring to desired degree of precision.
	MA.3.4.9	role-play making change up to $10.00.			MA.5.4.9	determine actual measurement from scale drawings.
	MA.3.4.10	estimate, read, and recognize common temperatures of Celsius and Fahrenheit.				
Standard 5: Data Analysis and Probability (MA.S.5)	MA.3.5.1	collect data from observation, surveys, and experiments, and construct and label a graph.	MA.4.5.1	understand and reason about the use and misuse of statistics in our society.	MA.5.5.1	collect, organize, display, read and interpret data from a problem-solving situation in a stem and leaf plot.
	MA.3.5.2	use a timeline to determine a sequence of events.	MA.4.5.2	read and interpret information represented on a circle graph.	MA.5.5.2	identify probabilities and solve problems involving the probability of an event by using tree diagrams or by construction of a sample space representing all possible results.

Content Standards	Performance Indicators					
	Grade 3		Grade 4		Grade 5	
	MA.3.5.3	experiment and describe concepts of probability and chance and list possible outcomes from a sampling.	MA.4.5.3	collect, organize, display, read and interpret data from a problem solving situation in line graphs, bar graphs, tally charts and tables with scale increments greater than one.	MA.5.5.3	construct, read, and interpret tables, charts, and graphs to draw reasonable inferences or verify predictions.
	MA.3.5.4	analyze data represented on a graph using grade level appropriate questions.	MA.4.5.4	list all possible outcomes for an experiment using a tree diagram.	MA.5.5.4	carry out experiments to determine probability.
			MA.4.5.5	determine mean, median, mode and range from collected data.	MA.5.5.5	construct a circle graph.

Appendix D: West Virginia 2006 21st Century Mathematics Content Standards and Objectives (approved 11/15/2006, effective 7/1/2008)

Content Standards	Performance Indicators					
	Grade 3		Grade 4		Grade 5	
Standard 1: Number and Operations (M.S.3.1; M.S.4.1; M.S.5.1)	M.O.3.1.1	read, write, order, and compare numbers to 10,000 using a variety of strategies (e.g., symbols, manipulatives, number line).	M.O.4.1.1	read, write, order, and compare whole numbers to the millions place and decimals to thousandths place using a variety of strategies (e.g. symbols, manipulatives, number line, pictorial representations).	M.O.5.1.1	read, write, order and compare all whole numbers, fractions, mixed numbers and decimals using multiple strategies (e.g., symbols, manipulatives, number line).
	M.O.3.1.2	read, write, order, and compare decimals to hundredths, with manipulatives.	M.O.4.1.2	demonstrate an understanding of the place value of each digit utilizing standard and expanded form through 1,000,000 with multiples of 10 [(5 X 10,000) + (3 X 1,000) + (4 X 10) + 2].	M.O.5.1.2	demonstrate an understanding of place value of each digit utilizing standard and expanded form in any whole number using powers of 10 [(3 X 10^5) + (4 X 10^3) + 7 X 10^2) + (1 X 10^1) + 6].
	M.O.3.1.3	identify place value of each digit utilizing standard and expanded form to 10,000.	M.O.4.1.3	estimate solutions to problems including rounding, benchmarks, compatible numbers and evaluate the reasonableness of the solution, justify results.	M.O.5.1.3	estimate solutions to problems involving whole numbers, decimals, fractions, and percents to determine reasonableness using benchmarks.

Content Standards	Performance Indicators					
	Grade 3		Grade 4		Grade 5	
	M.O.3.1.4	apply estimation skills (rounding, benchmarks, compatible numbers) to evaluate reasonableness of an answer.	M.O.4.1.4	using concrete models, benchmark fractions, number line compare and order fractions with like and unlike denominators add and subtract fractions with like and unlike denominators model equivalent fractions model addition and subtraction of mixed numbers with and without regrouping.	M.O.5.1.4	use inductive reasoning to identify the divisibility rules of 2, 3, 5, 9 and 10 and apply the rules to solve application problems.
	M.O.3.1.5	demonstrate an understanding of fractions as part of a whole/one and as part of a set/group using models and pictorial representations.	M.O.4.1.5	analyze the relationship of fractions to decimals using concrete objects and pictorial representations.	M.O.5.1.5	determine and apply greatest common factor and lowest common multiple to write equivalent fractions and to real-world problem situations.
	M.O.3.1.6	create concrete models and pictorial representations to compare and order fractions with like and unlike denominators, add and subtract fractions with like denominators, and verify results.	M.O.4.1.6	round decimals to the nearest whole, 10th, or 100th place.	M.O.5.1.6	model and write equivalencies of fractions decimals, percents, and ratios.

Content Standards	Performance Indicators					
	Grade 3		Grade 4		Grade 5	
	M.O.3.1.7	use concrete models to demonstrate an understanding of equivalent fractions, proper and improper fractions, and mixed numbers.	M.O.4.1.7	add and subtract whole numbers(up to five –digit number) and decimals to the 1000th place, multiply (up to three digits by two-digits, and divide(up to a three digit number with a one and two-digit number) .	M.O.5.1.7	analyze and solve application problems and justify reasonableness of solution in problems involving addition and subtraction of: fractions and mixed numbers decimals.
	M.O.3.1.8	add and subtract 2- and 3-digit whole numbers and money with and without regrouping.	M.O.4.1.8	solve multi-digit whole number multiplication problems using a variety of strategies, including the standard algorithm, justify methods used.	M.O.5.1.8	apply the distributive property as it relates to multiplication over addition.
	M.O.3.1.9	demonstrate and model multiplication (repeated addition, arrays) and division (repeated subtraction, partitioning).	M.O.4.1.9	quick recall of basic multiplication facts and corresponding division facts.	M.O.5.1.9	solve multi-digit whole number division problems using a variety of strategies, including the standard algorithm and justify the solutions.
	M.O.3.1.10	use and explain the operations of multiplication and division including the properties (e.g., identity element of multiplication, commutative property, property of zero, associative property, inverse operations).	M.O.4.1.10	create grade-level real-world appropriate story problems using multiple strategies including simple ratios, justify the reason for choosing a particular strategy and present results.	M.O.5.1.10	demonstrate fluency in addition, subtraction, multiplication and division of whole numbers.

Content Standards	Performance Indicators					
	Grade 3		Grade 4		Grade 5	
	M.O.3.1.11	recall basic multiplication facts and the corresponding division facts.			M.O.5.1.11	solve real-world problems involving whole numbers, decimals and fractions using multiple strategies and justify the reasonableness by estimation.
	M.O.3.1.12	model the distributive property in multiplication of 2- and 3-digit numbers by a 1-digit number.				
	M.O.3.1.13	use models to demonstrate division of 2- and 3-digit numbers by a 1-digit number.				
	M.O.3.1.14	create grade-appropriate real-world problems involving any of the four operations using multiple strategies, explain the reasoning used, and justify the procedures selected when presenting solutions.				
Standard 2: Algebra (M.S.3.2; M.S.4.2; M.S.5.2)	M.O.3.2.1	analyze and extend geometric and numeric patterns.	M.O.4.2.1	determine the rule and explain how change in one variable relates to the change in the second variable, given an input/output model using two operations.	M.O.5.2.1	use inductive reasoning to find missing elements in a variety of patterns (e.g., square numbers, arithmetic sequences).
	M.O.3.2.2	create an input/output model using addition, subtraction, multiplication or division.	M.O.4.2.2	recognize and describe relationships in which quantities change proportionally.	M.O.5.2.2	given an input/output model using two operations, determine the rule, output or input.

Content Standards	Performance Indicators					
	Grade 3		Grade 4		Grade 5	
	M.O.3.2.3	analyze a given pattern and write the rule.	M.O.4.2.3	represent the idea of a variable as an unknown quantity using a letter, write an expression using a variable to describe a real-world situation.	M.O.5.2.3	solve simple equations and inequalities using patterns and models of real-world situations, create graphs of the equations and interpret the results.
	M.O.3.2.4	write equivalent numerical expressions and justify equivalency.	M.O.4.2.4	solve real-world problems involving order of operations including grouping symbols and the four operations,	M.O.5.2.4	model identify and describe square, prime and composite numbers.
	M.O.3.2.5	use symbol and letter variables to represent an unknown quantity and determine the value of the variable.				
Standard 3: Geometry (M.S.3.3; M.S.4.3; M.S.5.3)	M.O.3.3.1	identify and create new polygons by transforming, combining and decomposing polygons.	M.O.4.3.1	identify, classify, compare and contrast two-dimensional (including quadrilateral shapes) and three-dimensional geometric figures according to attributes.	M.O.5.3.1	classify and compare triangles by sides and angles; measure the angles of a triangle using a protractor.

Content Standards	Performance Indicators					
	Grade 3		Grade 4		Grade 5	
	M.O.3.3.2	identify, describe, and classify the following geometric solids according to the number of faces, edges, and vertices: cube rectangular solid cylinder cone pyramid	M.O.4.3.2	recognize and describe three-dimensional objects from different perspectives.	M.O.5.3.2	construct and analyze three-dimensional shapes using properties (i.e. edges, faces or vertices).
	M.O.3.3.3	construct and identify a solid figure from a plane drawing.	M.O.4.3.3	identify, draw, label, compare and classify lines (intersecting, parallel, and perpendicular) angles (acute, right, obtuse, and straight)	M.O.5.3.3	create a design with more than one line of symmetry.
	M.O.3.3.4	identify, describe and draw lines of symmetry in two-dimensional shapes.	M.O.4.3.4	identify and create a two-dimensional design with one line of symmetry.	M.O.5.3.4	construct a circle with a given radius or diameter.
	M.O.3.3.5	model, describe, and draw lines rays angles including right, obtuse, and acute angles.	M.O.4.3.5	graph/plot ordered pairs on a first-quadrant grid and use the coordinate system to specify location and describe path.	M.O.5.3.5	draw a similar figure using a scale, given a real-world situation.

Content Standards	Performance Indicators					
	Grade 3		Grade 4		Grade 5	
	M.O.3.3.6	draw an example of a flip, slide and turn (reflection, translation, and rotation) given a model.	M.O.4.3.6	draw and identify parts of a circle: center point, diameter, and radius.		
	M.O.3.3.7	name the location of a point on a first-quadrant grid, represent using ordered pairs.	M.O.4.3.7	select, analyze and justify appropriate use of transformations (translations, rotations, flips) to solve geometric problems including congruency and tiling (tessellations).		
Standard 4: Measurement (M.S.3.4; M.S.4.4; M.S.5.4)	M.O.3.4.1	estimate, measure, compare, and order common measurements of objects: length using customary and metric (to the nearest 1/2 inch) temperature in Celsius and Fahrenheit mass/weight	M.O.4.4.1	select appropriate measuring tools, apply and convert standard units within a system to estimate, measure, compare and order real-world measurements including: lengths using customary (to the nearest one-fourth inch) and metric units, weight, capacity, temperature, and justify and present results.	M.O.5.4.1	estimate, measure, compare, order and draw lengths of real objects in parts of an inch up to 1/8 of an inch and millimeters.
	M.O.3.4.2	estimate and find the perimeter and area of familiar geometric shapes, using manipulatives, grids, or appropriate measuring tools.	M.O.4.4.2	demonstrate an understanding of the formula used to determine the area of rectangles and squares and use this formula to compare areas of rectangles and squares.	M.O.5.4.2	model, calculate and compare area of triangles and parallelograms using multiples strategies (including, but not limited to, formulas).

Content Standards	Performance Indicators					
	Grade 3		Grade 4		Grade 5	
	M.O.3.4.3	determine the formula the area of a rectangle and explain reasoning through modeling.	M.O.4.4.3	read time to the minute, calculate elapsed time in hours/minutes within a 24-hour period.	M.O.5.4.3	develop strategies (i.e. finding number of same sized units of volume)to determine the volume of a rectangular prism; solve application problems involving estimating or measuring volume of rectangular prisms.
	M.O.3.4.4	read time to 5-minute intervals using analog and digital clocks, compute elapsed time to the quarter-hour using a clock.	M.O.4.4.4	given real-world situations, count coins and bills and determine correct change.	M.O.5.4.4	describe the effects on the measurements of a two-dimensional shape (such as its perimeter and area) when the shape is changed in some way, justify changes.
	M.O.3.4.5	identify, count and organize coins and bills to display a variety of price values from real-life examples with a total value of $100 or less and model making change using manipulatives.			M.O.5.4.5	solve real-world problems requiring conversions within a system of measurement.
					M.O.5.4.6	estimate and/or measure the weight/mass of real objects in ounces, pounds, grams, and kilograms.
					M.O.5.4.7	collect, record, estimate and calculate elapsed times from real-world situations (with and without technology)

Content Standards	Performance Indicators					
	Grade 3		Grade 4		Grade 5	
					M.O.5.4.8	determine the actual measurements of a figure from a scale drawing, using multiple strategies.
Standard 5: Data Analysis and Probability (M.S.3.5)	M.O.3.5.1	collect and organize grade-appropriate real-world data from observation, surveys, and experiments, and identify and construct appropriate ways to display data.	M.O.4.5.1	read and interpret information represented on a circle graph.	M.O.5.5.1	construct a sample space to predict the probability of a real-world simulation and test the prediction with experimentation.
	M.O.3.5.2	develop and conduct grade-appropriate experiments using concrete objects (e.g. counters, number cubes, spinners) to determine the likeliness of events and list all outcomes.	M.O.4.5.2	pose a grade-appropriate question that can be addressed with data, collect, organize, display, and analyze data in order to answer the question.	M.O.5.5.2	construct, read, and interpret tables, charts, and graphs including stem and leaf plots to draw reasonable inferences or verify predictions.
	M.O.3.5.3	analyze real-world data represented on a graph using grade-appropriate questions.	M.O.4.5.3	design and conduct a simple probability experiment using concrete objects, examine and list all possible combinations using a tree diagram, represent the outcomes as a ratio and present the results.	M.O.5.5.3	collect and organize real-world data to construct a circle graph (with and without technology), present data and draw conclusions.

Appendix E: NAEP Performance Descriptors by Grade Level Taught in WV 2006 Grade 3-5 Math Curriculum Standards

Content Area	Standards/ Skills	Total Number of Performance Descriptors (PD)	Number of PDs Taught in Grade 3 Only	Number of PDs Taught in Grade 4 Only	Number of PDs Taught in Grade 5 Only	Number of PDs Taught in Grades 3,4	Number of PDs Taught in Grades 3,4,5
Numbers and Properties	Number sense	6	2	0	2	0	2
	Estimation	3	0	0	0	0	3
	Number operations	6	0	0	1	1	4
	Ratios and proportional reasoning	1	0	0	0	1	0
	Properties of number and operations	4	0	1	0	0	3
Measurement	Measuring physical attrib- utes	6	0	0	1	2	3
	Systems of measurement	4	0	0	0	1	3
Geometry	Dimension and shape	4	0	0	3	0	1
	Transformation of shapes and preservation of prop- erties	4	0	0	0	0	4
	Relationships between geometric figures	4	0	0	1	0	3
	Position and direction	2	0	0	2	0	0

	Mathematical reasoning	1	0	0	1	0	0
Data Analysis and Probability	Data representation	3	0	0	3	0	0
	Characteristics of data sets	2	0	0	2	0	0
	Experiments and samples	0	0	0	0	0	0
	Probability	4	0	0	4	0	0
Content Area	Standards/ Skills	Total Number of Performance Descriptors (PD)	Number of PDs Taught in Grade 3 Only	Number of PDs Taught in Grade 4 Only	Number of PDs Taught in Grade 5 Only	Number of PDs Taught in Grades 3,4	Number of PDs Taught in Grades 3,4,5
Algebraic Rep-resentations	Patterns, relations, and functions	5	0	0	3	0	2
	Algebraic representations	3	0	0	3	0	0
	Variables, expressions, and operations	2	0	0	2	0	0
	Equations and inequalities	1	0	1	0	0	0
Total/Percent of Total		65	2	2	28	5	28